07 - 10 September 2009

PROCEEDINGS

54. IWK

Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

Information Technology and Electrical Engineering - Devices and Systems, Materials and Technologies for the Future

Faculty of Electrical Engineering and Information Technology

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c.
Peter Scharff

Redaktion: Referat Marketing
Andrea Schneider

Fakultät für Elektrotechnik und Informationstechnik
Univ.-Prof. Dr.-Ing. Frank Berger

Redaktionsschluss: 17. August 2009

Technische Realisierung (USB-Flash-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag:
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

ISBN (USB-Flash-Ausgabe): 978-3-938843-45-1
ISBN (Druckausgabe der Kurzfassungen): 978-3-938843-44-4

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

ABOUT DISTRIBUTIONS OF EDDY CURRENTS ON LAYER WITH HETEROGENEOUSE ANISOTROPIC CONDUCTIVITY

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ABSTRACT

The problem for the eddy currents computation in conductive layer in quasi stationary magnetic field is reduced to the integral-differential equation. The layer has heterogeneous anisotropic conductivity. The analysis of the equation is carried out by the variational method. The novel software package has been created for its solving. The examples of its usage are represented. The influence of anisotropic and heterogeneous conductive properties of the layer on the eddy currents distribution is considered.

Index Terms **–** Integral-differential equation, Eddy current, thin conductive layer, quasi stationary magnetic field

1. INTRODUCTION

Many engineering problems require the simulation of stationary and quasi-stationary magnetic fields in the presence of conductive layers (cases, plates). Such layers play a part of protective screens, frames and load-bearing elements of electro technical and electrical survey devices. At present time membranous and printing technologies are developed therefore the interest to such problems has grown.

In particular cases the problems of stationary magnetic field computation in presence of a conductive surface can be solved analytically. If the surface has complicated geometrical form and boundary then the most useful method for problem solving is the Integral Equation Method. This method has low cost computer realization. Using them, the spatial boundary problem for magnetic field computation can be reduced to surface integral-differential equation for scalar flow function of eddy currents density. This function is non zero only in conductive surface.

Also, in practice distribution of eddy currents depends on conductivity properties of material. Research of this dependence is very important.

In this paper the computation problem for the eddy currents of conductive layer with holes in quasi stationary magnetic field is reduced to the integral-differential equation. This layer has heterogeneous anisotropic conductivity. We prove that the problem is well defined in the special pair of functional spaces. These spaces are established by the physics properties of the problem. The numerical solving of the problem is carried out using Bubnov-Galerkin method. The influence of anisotropic and heterogeneous conductive properties of the layer on the eddy currents distribution is considered.

2. PROBLEM STATEMENT

Consider a thin conductive layer with holes which is situated in quasi stationary magnetic field. The layer has thickness h and finite anisotropic heterogeneous conductivity γ.

Assume that the layer is thin, i.e., h is far smaller than other dimensions of the layer. In this case, the normal component of eddy currents density and its thickness nonuniformity are unimportant.

Let Γ' be median surface of the layer with piecewise smooth boundary $\partial\Gamma''$. Also, Γ' has a finite number $N+1$ of holes Γ''_0, $\Gamma''_1,...,\Gamma''_N$, denote their union by Γ''. The median surface has exterior normal $\mathbf{n}$.

Consider a closed surface $\Gamma = \Gamma' \bigcup \Gamma'' \bigcup \partial\Gamma''$. Introduce orthogonal coordinate system $\{0,u,v,w\}$ with unit vectors $\mathbf{e}_u, \mathbf{e}_v, \mathbf{n}$. Zero point of w-coordinate is situated on the surface Γ (see fig. 1).

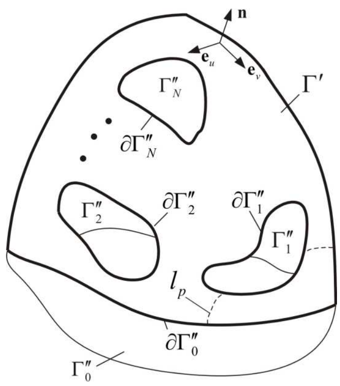

Figure 1 Problem statement

Further, by $\boldsymbol{\sigma}$ denote linear density of the eddy currents such that $\boldsymbol{\sigma} = \int_{-h/2}^{h/2} \boldsymbol{\delta} dn$. Here $\boldsymbol{\delta}$ is the current density in the conductive layer. The conductivity of the surface Γ' is described by tensor of the linear conductivity

$$\gamma_s = \begin{Vmatrix} \gamma_{uu} & 0 \\ 0 & \gamma_{vv} \end{Vmatrix}.$$

Its elements are bounded and pieciwise smooth functions of variable u and v. At that we consider linear conductivity as product of the conductivity γ and thickness of the layer.

Let's non-perturbed magnetic field (or equivalent for it in terms of [1]) is given by sources that are located outside Γ' in a bounded domain of space. This field has finite energy, i.e.,

$$\int_\Omega \left|\mathbf{B}^0\right|^2 d\Omega < \infty .$$

Suppose the external medium is homogeneous with a finite positive magnetic conductivity.

The computation of the eddy currents on Γ' is reduced to boundary value problem. By introduced definition this problem in steady state mode has the following form

$$\mathrm{rot}_n\left(\gamma_s^{-1}\dot{\boldsymbol{\sigma}}\right) = -j\omega\dot{B}_n \text{ on } \Gamma'; \tag{1}$$

$$\mathrm{div}\,\dot{\boldsymbol{\sigma}} = 0 \text{ on } \Gamma'; \tag{2}$$

$$\mathrm{Div}\,\dot{\boldsymbol{\sigma}} = 0 \text{ on } \partial\Gamma''; \tag{3}$$

$$\dot{\boldsymbol{\sigma}} = 0 \text{ on } \Gamma'', \tag{4}$$

where variables with point are complex amplitudes of respective variables; ω is an angular frequency; $j = \sqrt{-1}$. Here Maxwell equations, constitutive equation and boundary conditions for the magnetic field [2] are used.

Lines l_p are discontinuity lines for elements of the tensor γ_s. On these lines Faraday's law of electromagnetic induction (1) and condition of solenoidal (2) are replaced by conditions of conjugation. They have the following form

$$\left[\left(\gamma_s^{-1}\dot{\boldsymbol{\sigma}}\right)^+ - \left(\gamma_s^{-1}\dot{\boldsymbol{\sigma}}\right)^-, \mathbf{v}\right] = 0, \ \left(\dot{\boldsymbol{\sigma}}^+ - \dot{\boldsymbol{\sigma}}^-, \mathbf{v}\right) = 0. \tag{5}$$

Here $\mathbf{v}$ is normal to l_p. It is situated in tangent plane to the surface Γ. Symbols "$+$" and "$-$" in superscript denote the limiting values along positive and negative directions of the normal $\mathbf{v}$ respectively.

Since elements of the tensor γ_s is equal to zero on Γ'', vector $\left(\gamma_s^{-1}\dot{\boldsymbol{\sigma}}\right)^-$ is uncertain quantity on $\partial\Gamma''$. Therefore it is necessary to add subsidiary condition, which has the following form

$$\oint_{\partial\Gamma''}\left(\gamma_s^{-1}\dot{\boldsymbol{\sigma}}\right)^- dl = -j\omega\int_{\Gamma''}\dot{B}_n d\Gamma . \tag{6}$$

Induction $\mathbf{B}$ of the magnetic field can be represented in the form

$$\mathbf{B}(M) = \mathbf{B}^0(M) + \mathrm{rot}\frac{\mu_0}{4\pi}\int_{\Gamma'}\frac{\boldsymbol{\sigma}(Q)}{r_{QM}}d\Gamma_Q , \tag{7}$$

where $\mathbf{B}^0$ is the induction of the non-perturbed magnetic field; the induction of the magnetic reaction field is expressed by the eddy currents density $\boldsymbol{\sigma}$ using Biot-Savart law [2].

Field of the eddy currents density $\boldsymbol{\sigma}$ is solenoidal. Hence we can introduce a scalar flow function τ for this density as following

$$\boldsymbol{\sigma} = \left[\mathrm{grad}\,\tau, \mathbf{n}\right] \tag{8}$$

Then formula (7) is transformed to

$$\mathbf{B}(M) = \mathbf{B}^0(M) - \mathrm{grad}_M\frac{\mu_0}{4\pi}\int_{\Gamma'}\tau(Q)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}d\Gamma_Q .$$

After the boundary problem (1)-(6) is reduced to the integral-differential equation

$$\mathrm{div}_s\left(\tilde{\gamma}_s^{-1}\,\mathrm{grad}\,\dot{\tau}\right) = j\omega\dot{B}_n^0 - j\omega\frac{\partial}{\partial n_M}\frac{\mu_0}{4\pi}\int_{\Gamma'}\dot{\tau}(Q)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}d\Gamma_Q \quad \text{on } \Gamma' \tag{9}$$

with conditions

$$\dot{\tau}^+ = \dot{\tau}^-, \ \mathbf{v}\left(\tilde{\gamma}_s^{-1}\,\mathrm{grad}\,\dot{\tau}\right)^+ = \mathbf{v}\left(\tilde{\gamma}_s^{-1}\,\mathrm{grad}\,\dot{\tau}\right)^- \text{ on } l_p;$$

$$\dot{\tau} = c_k \text{ on } \Gamma_k'' \cup \partial\Gamma_k'', \ c_k = const, \ k = \overline{0,N}; \tag{10}$$

$$\oint_{\partial\Gamma''}\mathbf{v}\left(\tilde{\gamma}_s^{-1}\,\mathrm{grad}\,\dot{\tau}\right)^- dl = -j\omega\int_{\Gamma''}\dot{B}_n d\Gamma .$$

Here $\tilde{\gamma}_s = \begin{Vmatrix} \gamma_{vv} & 0 \\ 0 & \gamma_{uu} \end{Vmatrix}$. From (8) it follows that τ will be defined with accuracy to the arbitrary constant. In the sequel, this property can be used suitably.

3. ANALYSIS OF EQUATION

Now we analyze equation (9) by the variational method [3], i.e., we reduce the equation to an equivalent variational problem for some functional. Using Riss's theorem [3], we establish the variational problem as well defined. The variational method is useful, because effective way of the variational problem solving is well known. Here we use Bubnov-Galerkin's Method for the problem solving.

We have to extend equation (9) to the closed surface Γ for further analysis. Introduce the following operators

$$P\xi = \begin{cases} \xi & \text{on } \Gamma' \\ \dfrac{1}{mes\left(\Gamma_k''\right)}\displaystyle\int_{\Gamma_k''}\xi d\Gamma, & k = \overline{0,N}, \end{cases}$$

$$D\xi(M) = -\mathrm{div}_s\left(\tilde{\gamma}_s^{-1}\,\mathrm{grad}\,\xi(M)\right),$$

$$K_{\tau\sigma}\xi(M) = -\frac{1}{2\pi}\frac{\partial}{\partial n_M}\int_\Gamma \xi(Q)\frac{\partial}{\partial n_Q}\frac{1}{r_{QM}}d\Gamma_Q, \quad M \to \Gamma .$$

After that equation (9) and conditions (10) are reduced to unified operator equation

$$A_\tau \tau = f_\tau , \tag{11}$$

where

$$A_\tau = P\left(D + \frac{j\omega\mu_0}{2}K_{\tau\sigma}\right), \tag{12}$$

$$f_\tau(M)=\begin{cases} -\dot{B}_n^0(M), & M\in\Gamma', \\ -\dfrac{1}{mes(\Gamma''_k)}\displaystyle\int_{\Gamma''_k}\dot{B}_n^0 d\Gamma, & M\in\Gamma''_k,\ k=\overline{0,N}. \end{cases} \quad (13)$$

Moreover, the following condition is carried out

$$\int_\Gamma f_\tau d\Gamma=0. \quad (14)$$

We choose $\tilde{L}_2(\Gamma)$ as initial space for the analysis of the equation. It is Hilbert space of square-integrable functions with constant values on $\Gamma''_k,\ k=\overline{0,N}$. In this space the inner product and the norm are defined by

$$(a_1,a_2)_{\tilde{L}_2}=\int_\Gamma a_1\bar{a}_2 d\Gamma,\ \|a\|_{\tilde{L}_2}=(a,a)_{\tilde{L}_2}^{1/2}.$$

Since the operator A_τ is a linear combination of two operators, then consider those operators singly. Denote they by

$$A_\tau^{(1)}=PD \text{ and } A_\tau^{(2)}=PK_{\tau\sigma}.$$

The operator $A_\tau^{(2)}$ is linear, self-adjoint and positive in $\tilde{L}_2(\Gamma)$ according to [4]. But it is necessary to add subsidiary condition for τ. For example, the condition is similar to (14). This calibration allows to choose the subspace $\tilde{L}_2^0(\Gamma)$ in $\tilde{L}_2(\Gamma)$. It consists of the elements of $\tilde{L}_2(\Gamma)$ with zero mean value on Γ. The received subspace can be used as the initial space for the variational problem.

Consider properties of the operator $A_\tau^{(1)}$. The linearity of this operator is obvious. Prove its self-adjointness. Let's execute

$$\left(A_\tau^{(1)}\xi_1,\xi_2\right)_{\tilde{L}_2}=(PD\xi_1,\xi_2)_{\tilde{L}_2}=(D\xi_1,P\xi_2)_{\tilde{L}_2}=$$
$$=(D\xi_1,\xi_2)_{\tilde{L}_2}=-\int_\Gamma \operatorname{div}_s\left(\tilde{\gamma}_s^{-1}\operatorname{grad}\xi_1\right)\bar{\xi}_2 d\Gamma=$$
$$=\int_\Gamma \tilde{\gamma}_s^{-1}\operatorname{grad}\xi_1\operatorname{grad}\bar{\xi}_2 d\Gamma=(\xi_1,D\xi_2)_{\tilde{L}_2}=$$
$$=\left(\xi_1,A_\tau^{(1)}\xi_2\right)_{\tilde{L}_2}\ \forall\xi_1,\xi_2\in\tilde{L}_2^0(\Gamma),$$

where identities for vector analysis [5] are used.

Now consider

$$\left(A_\tau^{(1)}\xi,\xi\right)_{\tilde{L}_2}=\int_\Gamma\tilde{\gamma}_s^{-1}\operatorname{grad}\xi\operatorname{grad}\bar{\xi}d\Gamma\geq$$
$$\geq\frac{1}{\gamma_{s\min}}\int_\Gamma\operatorname{grad}\xi\operatorname{grad}\bar{\xi}d\Gamma\geq\frac{1}{\gamma_{s\min}d^2}\int_\Gamma\xi\bar{\xi}d\Gamma$$
$$\forall\xi\in\tilde{L}_2^0(\Gamma),$$

where $\gamma_{s\min}$ is least value for the conductivity of Γ, d is least value for diameter of the surface Γ, and Friedrichs inequality [3] are used.

Thus, the operator $A_\tau^{(1)}$ is linear, self-adjoint and positive defined in $\tilde{L}_2^0(\Gamma)$. Taking into account properties of the operators $A_\tau^{(1)}$ and $A_\tau^{(2)}$ in $\tilde{L}_2^0(\Gamma)$, we have the operator A_τ linear, self-adjoint, and positive defined in $\tilde{L}_2^0(\Gamma)$.

Using the properties of the operator A_τ in $\tilde{L}_2^0(\Gamma)$, we get that the equation (9) and the variational problem

$$F(\tau)=(A_\tau\tau,\tau)_{\tilde{L}_2}-2\operatorname{Re}(f_\tau,\tau)_{\tilde{L}_2}\to\min \quad (15)$$

are equivalent in the energetic space of this operator in $\tilde{L}_2^0(\Gamma)$. Denote the energetic space of the operator A_τ in $\tilde{L}_2^0(\Gamma)$ by $H_{A\tau}$. It is the Hilbert space of functions with constant values on $\Gamma''_k,\ k=\overline{0,N}$ and zero mean values on Γ. In this space the inner product and the norm are defined by

$$(\tau_1,\tau_2)_{H_{A\tau}}=(A_\tau\tau_1,\tau_2)_{\tilde{L}_2},\ \|\tau\|_{H_{A\tau}}=(\tau,\tau)_{H_{A\tau}}^{1/2}.$$

Taking into account positive definiteness of A_τ in $\tilde{L}_2^0(\Gamma)$, we have that the functional $G(\tau)=(f_\tau,\tau)_{\tilde{L}_2}$ is bounded in $H_{A\tau}$ for the given problem statement. Further, using Riss's theorem we conclude that variational problem (15) and equation (9) are both solved uniquely in $H_{A\tau}$.

4. NUMERICAL SOLVING OF EQUATION

The numerical solving of the variational problem is carried out using Bubnov-Galerkin method [6]. The solution is presented as following

$$\dot{\tau}(M)=\sum_{i=1}^n\dot{c}_i\tau_i(M),\ \tau_i\in\tilde{L}_{2A\tau}(\Gamma).$$

The coefficients $\{\dot{c}_i\}_{i=1}^n$ are defined by solving the following SLAE (system of linear algebraic equations)

$$\sum_{i=1}^n\dot{c}_i(\alpha_{ik}+j\omega\beta_{ik})=j\omega\dot{f}_{ik},\ k=\overline{1,n}. \quad (16)$$

The elements of the system are defined by the following formulas

$$\alpha_{ik}=\int_\Gamma\tilde{\gamma}_s^{-1}\operatorname{grad}\tau_i\operatorname{grad}\tau_k d\Gamma,\ i,k=\overline{1,n},$$

$$\beta_{ik}=-\frac{\mu_0}{4\pi}\int_{\Gamma_i}\int_{\Gamma_k}\frac{\operatorname{grad}\tau_k(Q)}{r_{QM}}d\Gamma_Q\operatorname{grad}\tau_i(M)d\Gamma_M,$$
$$i,k=\overline{1,n},$$

$$\dot{f}_k=\int_\Gamma\dot{B}_n^0\tau_k d\Gamma,\ k=\overline{1,n}.$$

The coordinate functions $\tau_i,\ i=\overline{1,n}$ can be taken from a class of continuous and piecewise continuous differentiable on Γ functions. For example, it can be piecewise-polynomial functions which are shown in fig. 2.

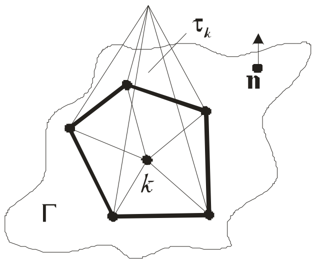

Figure 2 Example of coordinate function

In this case we have calculation formulas for the elements of the main matrix of SLAE (16) such that the most part of this calculation are analytically.

The novel software package for the numerical realization of the developed theory has been created by using Microsoft Visual C# 2008 programming language. This package allows to compute the eddy currents on surfaces with various geometrical forms. The result of computation can be represented as numerical date or as pictures of the eddy currents distribution.

The eddy currents distributions on the conductive plate in homogeneous cross magnetic field at different values of the parameter $\mu_0\gamma_x h\omega a$ are represented in fig. 3. Here a is one of plate's linear size, the plate is produced from material with anisotropic conductivity, where $\gamma_y = 10\gamma_x$.

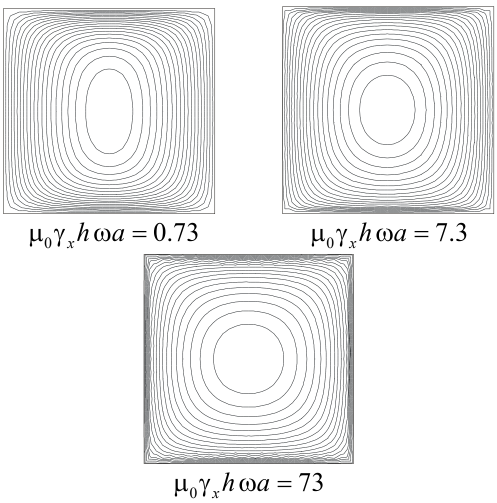

Figure 3 Eddy currents distribution on plate with anisotropic conductivity

The eddy currents distributions on the plate with heteregeneous conductivity in this field are represented in fig. 4. The plate consists of two parts, those are produced from materials with conductivities γ_1 and γ_2 $(\gamma_1 > \gamma_2)$.

Dependences of the flow function τ on the coordinate x are shown in fig. 5. These dependences are calculated on medial line for square conductive plate. Curves 1 and 2 are obtained for plates from previous examples (fig. 3 and fig. 4) respectively, curve 3 is calculated for square plate with the same geometry and isotropic homogeneous conductivity γ. Besides,

- $\tilde{\gamma} = \gamma_x$ for the curve 1,
- $\tilde{\gamma} = \gamma_1$ for the curve 2,
- $\tilde{\gamma} = \gamma$ for the curve 3.

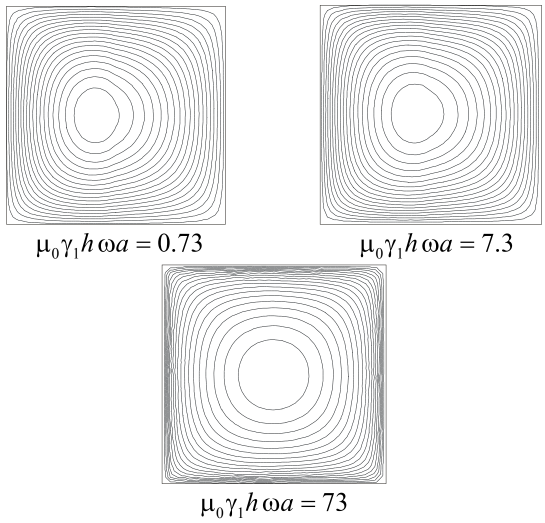

Figure 4 Eddy currents distribution on plate with heterogeneous conductivity

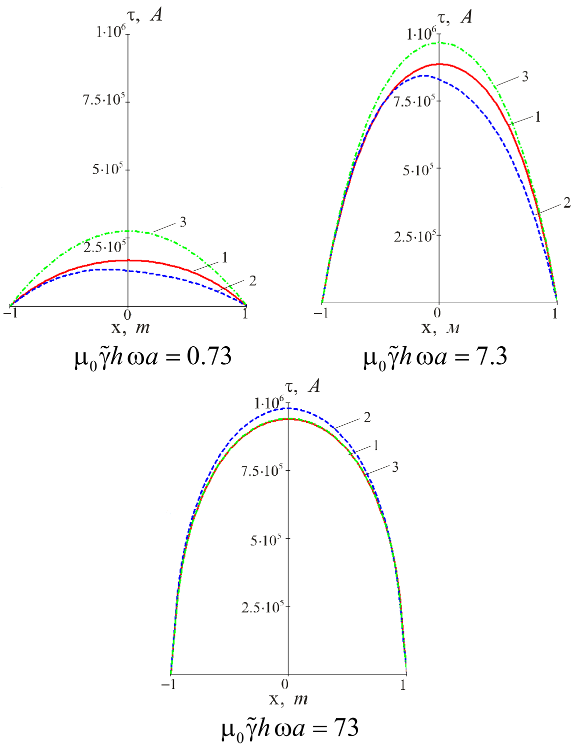

Figure 5 Dependence of the flow function τ on coordinate x for plate with anisotropic (1), heterogeneous (2), homogeneous (3) conductivity

It is obvious the anisotropy and heterogeneity of conductive properties of material take effect on the eddy currents distribution, but their influence decrease if the parameter $\mu_0\tilde{\gamma}h\,\omega a$ grows up. In the finally this influence is reduced to absolutely null.

5. CONCLUSION

The mathematical model has been created for the eddy currents computation in layer with holes and anisotropic heterogeneous conductivity in quasi stationary magnetic field. The model is based on the Integral Equation Method and it has low cost computer realization. Using this method the spatial boundary problem is transformed to the surface problem for steady state operation mode that is well defined in the spatial pair of functional spaces. For numerical realization of the developed theory the novel software package has been created using Microsoft Visual C# 2008 programming language. In this paper the examples of its of its usage are represented. Using this figure we can conclude that the anisotropy and heterogeneity of conductive material properties take effect on the eddy currents distribution. But this influence dependences on many factors. In particular if the angular frequency ω is grown up the influence of conductive material properties reduced to absolutely null.

Here we have considered only steady state operation mode. In the sequel, we plane to consider this problem for transient operation mode.

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